

- Working at equal rates of speed, 8 men take 12 days to finish a job. How long will it take one man to do the job?

men ×	days ÷	jobs	man-days/job
8	12	1	96
1	x=96	1	96

We immediately see that  $x = 96$ .

- One man works 10 days to finish a job. How long will it take five men to do the same job, all working at equal rates of speed?

men ×	days ÷	jobs	man-days/job
1	10	1	10
5	x=2	1	10

We see immediately that  $x = 2$ .

- How long will take 8 men to do a job that is done by 12 men in 40 days, working at equal rates of speed?

men ×	days ÷	jobs	man-days/job
12	40	1	480
8	x=60	1	480

Therefore 8 men take 60 days.

- If 3 men do a job in 12 days and two of the men are three times as fast as the third, how long will it take one of the faster men to do the job?

The men work in the ratios 3:3:1. That is, the two fast men each do  $\frac{3}{7}$  of the job in 12 days while the slow man does  $\frac{1}{7}$  of the job in 12 days. This can be visualized in the following table. One fast man working alone can do the job in  $12/(\frac{3}{7}) = 4 \times 7 = 28$  days.

	men ×	days ÷	jobs	man-days/job
	3	12	1	36
a	1	12	$\frac{3}{7}$	$\frac{84}{3} = 28$
b	1	12	$\frac{3}{7}$	$\frac{84}{3} = 28$
c	1	12	$\frac{1}{7}$	$\frac{84}{1} = 84$

Another way to view the problem is look at the number of man-days in terms of the speed of the slowest man. They work in the ratio 3:3:1, therefore  $3+3+1 = 7$  is the number of equal man-days based upon the speed of the slowest man. Since the three men work 12 days, they work  $7 \times 12 = 84$  man-days in units of the slowest man. The

slowest man working alone would take 84/1 days to the job. One of the faster men working alone would take 84/3 days. This is illustrated in the following table.

	men ×	days ÷	jobs	man-days/job
	3	12	1	36
a	1	$x=84/3=28$	1	84/3
b	1	x	1	84/3
c	1	x	1	84/1

Notice that it becomes easy to answer the question of how long would it take a fast man working with the slow man to do the job. It would be 84/4 or 21 days.

Now let’s invert this whole problem and pretend we are asked the following question:

*If three men working alone can complete a task in 28, 28, and 84 days, respectively, how long will it take them working together?*

In tabular form, this would look like:

	men ×	days ÷	jobs	man-days/job
a	1	1	1/28	28
b	1	1	1/28	28
c	1	1	1/84	84
			$1/28+1/28+1/84 = 1/12$	
	3	12	1	36

Adding the fractions of the job completed in one day gives 1/12 of the job in one day; that is, it would take 12 man-days to complete the job with all three men working together. Since there are 3 men working, they can complete the job in 12 days. This takes us full circle and illustrates both sides of the “work” problem.

5. If it takes a man 12 days to do a job, how much of the work will he do in 3 days?

	men ×	days ÷	jobs	man-days/job
	1	12	1	12
	1	3	$x = 1/4$	12

X must equal 1/4 to complete the table.

6. If 5/8 of a job is done in 15 days, how long will it take to complete the job?

	men ×	days ÷	jobs	man-days/job
	1	15	5/8	$x = 15/(5/8) = 24$

X must equal 24 to complete the table.

7. If it takes man A 4 days to build a boat, B 6 days for the same job, and C 10 days, how long will it take them to do the job working together?

We must reconstitute the table with days = 1 so that we can get the fraction of the whole job that each worker can do in one day. If you recognize the problem you can just write down the reciprocals of the days, add them, and invert.

	men ×	days ÷	jobs	man-days/job
A	1	1	1/4	4
B	1	1	1/6	6
C	1	1	1/10	10
			1/4+1/6+1/10 = 31/60	
T	3	60/31	1	180/31 = 5.8

We must add up these fractional jobs and take the reciprocal.

$$1/4 + 1/6 + 1/10 = (15 + 10 + 6)/60 = 31/60, \text{ so } x = 60/31.$$

8. A can do a job in 5 days and B in 10 days. A, B, and C working together can do it in 2 days. How long will it take C to do it by himself?

	men ×	days ÷	jobs	man-days/job
A	1	1	1/5 = 2/10	5
B	1	1	1/10	10
C	1	1	(a) 2/10	(b) 5
C	1	5	1	5
			1/2 = 5/10	
T	3	2	1	6

We must solve  $\frac{2}{10} + \frac{1}{10} + x = \frac{5}{10}$   $x = 2/10 = 1/5$ , which is 1/5 of the job in one day or 1 job in 5 days.