## Restaurant Bill - The process of adding a percentage to a base amount

Let's take a look at the seemly simple task of computing the total cost of eating out when the cost of the meal includes both tax and tip. We all know that the total cost can be written as a simple sum of three quantities as follows:

$$
\text { Total cost }=\text { food }+ \text { tax }+ \text { tip }
$$

But did you know that the mathematician would prefer to rewrite the total food cost equation as the product of three quantities in the following form.

$$
y=x(1+r)(1+s)
$$

where $x=$ cost of food, $r=$ sales tax rate, $s=$ tipping rate
What is this? How did they turn an addition problem into a multiplication problem? Isn't this what mathematicians do so well, namely, take a simple problem, recast it in a more generalized form, and make it nearly impossible for the non-mathematician to understand? Don't despair. As budding mathematicians, we will unmask this mathematical trickery and relish is our new skills

Let's return to the original total cost equation as the sum of three quantities and try to recast it as a mathematician might. First food cost is food cost, no surprises there. What about taxes? Taxes are not a fixed additive amount, but rather a percentage rate, which is really multiplicative. . That is, the tax amount can be written as the food cost " $x$ " multiplied by the tax rate " $r$ "

$$
\mathbf{x} \times \mathbf{r}
$$

We are using the multiplication sign " $\times$ " here just for emphasis. We know that when writing a mathematical formal consisting of single letters, we usually omit the " $x$ " and just write the letters side by side with the understanding that the multiplication is assumed when we do this. Now let's write the cost of the just the food and the tax, ignoring tip for the moment. The cost of food and tax can be written as

$$
\text { Cost without tip }=x+x r
$$

but we can rewrite this as

$$
x(1+r)
$$

which is the first of our multiplicative terms. This leads us to the following general rule:

## Percentage Rate Formula

When we need to add a percentage rate " $r$ " to a base amount " $x$ ", we can always recast the computation as the multiplication of the base amount by one plus the rate or

$$
\mathbf{x}(\mathbf{1}+\mathbf{r})
$$

We will pause here and do a simple example. In the Commonwealth of Virginia, the sales tax rate $4.5 \%$. On a $\$ 20$ restaurant bill the total tax would be

$$
\$ 20 \times 4.5 \%=\$ 20 \times .045=\$ 0.90=90 \text { cents }
$$

The total cost is

$$
\text { Food cost }+ \text { tax }=\$ 20.00+\$ 0.90=\$ 20.90
$$

Now we will repeat the computation using the percentage rate formula.

$$
\text { Food cost }=\$ 20.00 \times(1+.045)=20 \times(1.045)=\$ 20.90
$$

Take your calculator and check it for your self. Study this and learn it. This principle is used again and again and again in every day life, for example, interest rates, birth rates, death rates, income tax rates, sales tax rates, and tipping rates. Yes, even tipping rates, which we will now use to generate the third term in our equation.

Tips are also expressed as a percentage, commonly $15 \%$ or $20 \%$. We will use the letter "s" for the tip rate. Note that we can use whatever letter we want to represent the tipping rate; I just happened to choose "s" because it is the next letter in the alphabet after "r." Before we can write the formula, we have to decide if we will tip on just the food portion of the bill or on the bill amount including the tax. Assuming that we use the food bill including the tax to compute the tip, the tip amount is given by

$$
\text { Tip }=s \times \text { food bill }=s x(1+r)
$$

And the total cost is given by

$$
\begin{gathered}
\text { Cost }=\text { food bill }+ \text { tip }=x(1+r)+s x(1+r) \\
\text { Cost }=x(1+r)(1+s)
\end{gathered}
$$

which is just what we wanted to show in the first place.
Now you might ask, "how much extra am I tipping by tipping on the after tax bill rather than the before tax bill?" This is easily answered by multiplying out the terms in the above equation:

$$
\begin{gathered}
\text { Cost }=x+\mathbf{x r}+\mathbf{x s}+\mathbf{x r s} \\
\text { Cost }=\text { food }+ \text { tax }+ \text { tip }+ \text { over tip }
\end{gathered}
$$

Therefore, the extra amount is xrs.

If you normally tip $20 \%$ and the sales tax is $4.5 \%$ and your food bill is $\$ 20$ dollars on average, you over tip on average by $20 \times .045 \times .20=18$ cents, not hardly worrying about. What is the effective rate of over tipping? $.2 \times .045=.009$ or $.9 \%$. How much does the food cost have to be for you to over tip by $\$ 1$ ? This is found by solving

$$
\mathbf{r s x}=.009 \mathrm{x}=1
$$

which gives $\mathrm{x}=\$ 111.11$. So I guess we don't have to worry about excessively over tipping by tipping on the after portion of the bill. How might you modify the formula to exclude over tipping? We would have

$$
\operatorname{Cost}=x(1+r+s)
$$

What about tipping only on the cost of cost of the food, excluding wine? If $w$ is the cost of the wine, our new equation would be

$$
\operatorname{Cost}=(x-w)(1+r+s)
$$

