Mixture Problems from Arithmetic Refresher p339+

Problem 1. ....

A solution of 180 gallons contains 20% of A. What amount has to be added to increase the percent of A to 25%?

A	Not A	Total
20%		180
36	144	180
25%		
36+x		180+x
	144	(3/4)x

Method 1: Solve the equation

$\frac{36+x}{180+x} = .25$, where x is the amount to be added. x = 12 gallons

Method 2: Solve the equation

$144 = \frac{3}{4}x$, where x is the new total amount. x =192 gallons and the amount to be added is x-180. x = 190-180 = 12 gallons.

Problem 2. ....

How many quarts of water must added to 5 quarts of a 35% solution of hydrochloric acid to dilute it to a 25% solution?

The key is to focus on the ingredient that does not change and to set that equal to the new unknown total multiplied by its new percent. In this case, 2 gallons of water need to be added.

H	W	Total
.35		5
1.75	3.25	5
1.75		.25x = x/4
1.75	5.25	7

Problem 3. ....

Item A sells for $16 per pound, and item B sells for $24 per pound. How many pounds of A should be added to B to create 1000 pounds of a product that sells for $18.

Method 1: The problem can be viewed as a weighted average.

$\frac{16x + 24(1000 - x)}{1000} = 18$, where x is the amount of A. x = 750 pounds.

Method 2: The total new cost is equal to the sum of the cost of the components. This is just a minor variation of the weighted average approach.

$16x + 24(1000 - x) = 18 \times 1000$

Problem 4. ....

Three mixtures of alcohol consist of 6 gallons of 12% alcohol, 8 gallons of 14% alcohol, and 12 gallons of 35% alcohol. If all of the containers are combined into one container, what is the combined percent alcohol?

$$\frac{6(.12) + 8(.14) + 12(.35)}{6 + 8 + 12} = \frac{72 + 112 + 420}{2600} = \frac{604}{2600} = 23.23\%$$

Problem 5. ....

In what proportion should 45% alcohol mixture and 85% alcohol mixture be mixed to give a 68% mixture?

This is a problem in two unknowns of which we want to find the ratio. It can be set up as a weighted average as follows

$\frac{45x + 85y}{x + y} = 68$. Now we need to rearrange terms to find the ratio of x to y. $\frac{x}{y} = \frac{17}{23}$

That is, if we mix 17 volumes of x with 23 volumes of y, we will end up with a 68 percent solution of alcohol.

Problem 6. ....

How much water must be added to 1 pint of a solution of 25% antiseptic to reduce it to 15%?

This is confusing because the percents and the amounts get mixed up. 25% of 1 pint is .25 pints and after the dilution .25 pints = 15% x. x = 25/15 = 1.667 pints of total solution so 2/3 pint had to be added. Note that after the dilution, there is still .25 pints of antiseptic, but the new total is 1.667. 0.25/1.667 = 0.15.

Problem 7. ....

500 gallons of alcohol is 75% pure. How much water must be added to make it 40% pure?

The current solution has (3/4) 500 375 gallons of alcohol. After adding water, it will still have 375 gallons of alcohol, but the additional water will dilute it. 375 = .4x, x = 937.5. That is, after adding 437.5 gallons of water, the alcohol solution will be 40%. 375/937.5 = 40%.

Problem 8. ....

Products A and B are worth \$1.20 and \$1.60 per pound, respectively, How much of each product must be combined to make a new product of 50 pounds which is worth \$1.5 per pound?

This is a simple weighted average, $\frac{1.2(50-x)+1.6x}{50}=1.5$, x = 37.5. That is, we want to have 12.5 pounds of product A and 37.5 pounds of product B.

Problem 9. ....

How much cream which is 25% butterfat should be added to 1200 pounds of milk which is 3% butterfat to produce milk testing 4% butterfat?

The solution can be visualized two ways.

Method 1: The first approach starts by computing the current amount of butterfat in the 1200 pounds of milk, which is 36 = .03(1200). An equation which relates the pounds of butterfat after x pounds of a 25% solution is added can be written as:

$$36 + .25x = .04(1200 + x)$$

which gives x = 57 4/7.

Method 2: This method is a minor variation of the first method, where the problem is viewed as a weighted average.

$$\frac{.03(1200)+.25x}{1200+x}=.04$$

We can use this weighted average formulation to test the answer.

$$\frac{36+.25(57.1428)}{1257.1428}=\frac{36+14.2857}{1257.1428}=\frac{50.28}{1257.14}=.04$$

Problem 10. ....

One alloy is 25% silver and another is 40% silver. How much of each should be used to produce 60 pounds of an alloy that is 30% silver?

This is a straight forward weighted average.

$$\frac{.25(60-x)+.4x}{60}=.3\text{, x =20}$$

Add 20 pounds of 40% and 40 pounds of 25% to get 60 pounds of 30%.

Problem 11. ....

A confectioner wishes to mix chocolates worth 80 cents a pound with 40 pounds of bonbons worth 65 cents a pound to make a mixture worth 75 cents a pound. How many pounds of chocolates should he use?

This is a weighted average problem with one of the components known rather than the total.

$$\frac{.8(x-40)+.65(40)}{x}=.75\text{, x = 80 total pounds}$$

You will need to add 40 pounds of chocolates to create the 75 cent mixture.

Problem 12. ....

How much alcohol must be added to a mixture of 12 ounces of alcohol and 30 ounces of water to produce a mixture that is 60% alcohol?

Now the mixture is 12/42 = 2/7 alcohol and 30/42 = 5/7 water. We want A/T = 60% and 30/T = 40%. This one can be done fastest by focusing on the quantity that does not change, namely water. The amount of water before and after will remain the same. When you get done adding alcohol, you will still have 30 ounces of water, but it will represent 40% of the solution.

$.4x = 30$, x = 75, the new total of alcohol and water combined. The amount of alcohol is 75 – 30 = 45 and 45/75 = 60%.

Problem 13. ....

An automobile radiator holds 4 gallons and contains 25% antifreeze. How much of the solution must be drained from the radiator and antifreeze added to make the solution test 35% antifreeze?

This is a tricky problem. When you drain fluid, you are taking out both antifreeze and water in the ratio of 1 to 3. At present, the tank has 1 gallon of antifreeze mixed with 3 gallons of water for at total of 4 gallons. When we are done draining and adding pure antifreeze, we must have 1.4 gallons of antifreeze to generate the desired percent of 1.4/4 = 35%. If we drain an amount x, we can set up the following equation for the new amount of antifreeze in the solution

$$1 - .25x + x = 1.4$$

When drain, the alcohol will by .25 x since the current solution is 25% alcohol, and we will add pure alcohol to reach the desired amount of 1.4. This results in x, the amount to drain as .53 gallons. By draining .53 gallons, the alcohol will reduce by (.25)(.53) = .13 and the water by (.75)(.53) = .40. After draining .53 gallons, the new amounts of antifreeze and water are .87 and 2.6 = 3.47. Now .53 gallons of pure antifreeze will be added to bring the total fluid up to 4 galloons again. The water will be 2.6 gallons and the antifreeze will be 1.4 gallons. The antifreeze percent is 1.4/4 = 35%.