Arithmetic Problems

55 – The key word here is "**CANNOT**." In addition, when you see < rather than ≤, that is another clue. You should immediately notice that 12 is not allowed and see that 4×12 as not permissible.

58 – When ever you see **consecutive** positive integer always think n, n+1, n+2.

60 - The **smallest** sum will consist of the smallest primes you can find or the first three that you find greater than 20. Stop when you have found three. You don't need to consider even numbers since they can't be prime (the only even prime number is 2): 21, <u>23</u>, 25, 27, <u>29</u>, <u>31</u>. To add quickly begin by adding 29 and 31. It ends in 10. 29+31 = 60, 60+23 = 83.

89 – The key words here are "**must be true**." Scan for a good answer and try to avoid testing all the alternatives. This is basic algebra. You must remember that the only way ab or a/b can be positive is if a and b are either positive or both negative.

$$(+)\times(+)=(+)$$
$$(-)\times(-)=(+)$$
$$\frac{(+)}{(+)}=(+)$$
$$\frac{(-)}{(-)}=(+)$$

91 – When doing **approximations** always round to a **convenient number** to do the computation. Here you could round 5.98 to 6 or 5. Six is clearly more accurate than five, but five is easier and faster with respect to 600 and 15. Of course, if you go with six than you will also go with 16. In one case you end up with √240 and in the other with √200. From the choices it is easy to see that the answers lies between 5^2 = 25 and 20^2=400. In this problem, however you choose to round, you get you to the correct answer.

92 – Again the keyword "**CANNOT**." Look for the exception. Here, the answer must be (E) x+y. x plus anything cannot divide into x, so this CANNOT be true. The only way you will be able to jump to this answer is if you really understand gcd. Another way to think of gcd (x, y) is finding the largest value of k such that both

$$\frac{x}{k}\,and\,\frac{y}{k}$$

are integers.

100 – Remember that x ÷ 1/y = xy.

154 – There is a lot in this problem. For starters, we have to deal with the keyword "**CANNOT**", so we immediately look for obvious exceptions. The first thing to realize or visualize is that x/10=n or x=10n, which means x must be even, a multiple of 10, and, if x is positive, it must be greater than 10. At first glance, any number less than 10 wouldn't work, but we have to remember that **integers can be negative numbers**. So jumping on the obvious choice B would get us a wrong answer. (-

10/10 = -1, which is an integer!) The next obvious answer is E, an odd integer, which we know can't be true since 10n must be even. Just for educational purposes let's look at the other three choices. There are an infinite number of odd integers which when added together will give a number divisible by 10. Let n be odd, then n and k10-n are two odd integers, and (n + k10 – n) = k10 is divisible by 10. The product of two primes is a bit trickier since there is only one combination that will work, namely 5×2=10. Notice that **two is the only prime number that is even**. Since all other prime numbers are odd, and none of them are multiples of 10, they are not possibilities. There are an infinite number of consecutive integers which when added together will give a multiple of 10. They are special, but none the less, there are an infinite number. Notice than n + (n+1) + (n+2) = 3n + 3. You should just commit this to memory because it comes multiple times. See the discussion in 167 for more detail on sequences of consecutive integers. Now, if this is to be divisible by 10, we must have (3n+3)/10 = k an integer. Therefore, any integer n which satisfies the equation n = (10/3)k – 1 will work. That is, k has to be a multiple of three. These are all numbers that are one minus and one plus a multiple of 10, for example, 9+10+11, 19+20+21, etc.

164 – This is a "**least common multiple**" (lcm) problem. You need to find

lcm (1, 2, 3, 4, 5, 6, 7)

The prime factors are 2, , , , 2, 3, 7. Now select each prime number to its highest power and you get 3, 2^2, 5, 7. To multiply these numbers quickly, start with 4×5=20 since it ends in zero; it is easy to multiply again. Then multiply by 3 and finally by 7 to get 420. This can all be done in your head.

167 - The first thing to notice is "**consecutive integers**." This is the flag to think n, n+1, n+2. As with almost all these problems there is the straight forward approach and the "**see the trick**" approach. Unfortunately, the straight forward approach always takes longer than trick approach. The straight forward approach is to write down the sequence and solve for n. We get n + (n+1) + (n+2) + (n+3) + (n+4) = 5n + 10 = 560, n = 110. The next set of five numbers is (n+5) + (n+6) + (n+7) + (n+8) + (n+9) = 5n + 35 = 585. You add up the numbers 5+6+7+8+9 and get 35 or you can use formula for the sum of consecutive integers from F to L is equal to n(F+L)/2 = 5(9+5)/2) = 35. This is a trick in and of itself, which is useful in many situations, but it is not the main trick for the problem. Notice that we could have used this trick in the first place to find the equation 5n+10. The sum of any set of any k consecutive numbers beginning at n can be written as

$$\frac{k(n+(n+k-1))}{2} = \frac{k(F+L)}{2} = \frac{5(n+n+4)}{2} = 5n+10$$

where F and L are the first and last terms in the series. As neat as that trick is, it is not the main trick. The main trick is as follows. Notice that the differences between each pair of numbers which is five apart is five. That is

(n+5) – n = 5

(n+6) – (n+1) = 5
(n+7) – (n+2) = 5
(n+8) – (n+3) = 5
(n+9) – (n+4) = 5

Since there are five of these numbers five greater than the previous sequence of five numbers, the second set of five must be 5×5 larger than the first set of five, which gives a total for the second five of 560 + 25. Note that if there were two sequences of six integers, then the difference would be 6×6=36.

176 – This problem clearly requires a trick and you can assume any problem like it will require a similar trick. It is not obvious from the list of numbers that 1, 2, 3, 4 appears six times in each column, but you need to see this so that you and add 1+2+3+4=10 and multiply by 6 to get a column total of 60. Notice that the 24 integers comes from arranging 4 numbers in all permutations, which is 4!. This problem could easily be stated with any integer, e.g. 5. Of course, the higher the integer, the more they force you to find the trick. By keeping the problem small enough, they count on many people taking the brut force approach and wasting valuable seconds trying to actually add the numbers. This is hopeless.

184 – The obvious mistake here is to miss the fact that there is a zero between 5 and 7.

189 – Since both 5 and 7 are relatively prime, lcm(5, 7)=35. The best way to look at this is to find n such that $\frac{n}{5}$ *and* $\frac{n}{7}$ are integers, that is, 35k. Now the question asks, which of the following will divide into 35k, or which of the following is an integer:

$$\frac{35k}{12}, \frac{35k}{35}, \frac{35k}{70}$$

35k/70 is a candidate, but it is not an acceptable answer since it only works when k is even. The sucker choice is 35 and 70. Be careful.

251 – When you write down the number of the house and its corresponding color

1	2	3	4	5
W	G	W	G	W

You immediate notice that there one more white house than green houses. Seems easy, but there is a trap. This problem is characteristic of the errors which are made with edge or boundary conditions and how they are treated. The **sucker choice** is to immediate write n/2 + 1, when you should actually write (n+1)/2. Notice that n/2 + 1 = (n+2)/2, which is never an integer because n is odd! The book method seems way too long and complicated.

252- These problems are killers. If they really throw you, just guess and move on. You might test one possibility, but don't get bogged down. The first thing to do is to substitute letters for the symbols and the problem becomes:

$$\begin{array}{r} ab \\ \underline{\times\ ba} \\ a^2\ ab \\ \underline{ab\ b^2\ \ \ } \\ ab\ a^2{+}b^2\ ab \end{array}$$

You are given two key pieces of information $a \neq b$, and $ab<10$. Both are key. Since $ab<10$, there is no carrying when you do the multiplication, so ab = a. and b = 1. At this point it might be fastest to just start testing since 12 and 13 are eliminated. If you test 11, it will seem to work, but remember $a \neq b$, so this is the **sucker choice**. Test 21 it works so you are done. You can consult the book to see how you reason to b=2, but it is not worth the time if you can quickly test. This problem took me over 2 minutes the second time I did it. Don't get bogged down. This just might be a guess and go problem.

258 – This is a cute problem. Begin by writing down the sequence R G W BY and note that the necklace begins with R and ends with W. It shouldn't take too long to come up with the equation 5n + 3 for the total number of beads. Now you to have to test numbers. When you start to substitute values for n you get 8, 13, 18, and hopefully notice that the answer must end in 3 or 8, which allows you to select 68 immediately.

265 – The book method seems too long. Watch the edge conditions! The answer has to be less than 25, not equal to 25. The fastest method is to just write down values for n and corresponding values for 5n+5.

1	10
2	15
3	20
4	25

Note that 25 is not allowed and the answer is 3.

266 – This is a "**CANNOT**" problem and a classic candidate for the Sunday morning newspaper math puzzle – tricky but easy to understand when you get the trick.

a b	=	a10 + b
b a	=	b10 + a

When you add the two numbers you get 10(a+b) + (a+b) = 11(a+b). So whatever a and b are, the sum must be a multiple of 11. Since 181 is not a multiple of 11, it CANNOT be the answer. Consider 165, which is 11×15, a and b can be any two numbers which add to 15, for example, (7,8), (9,6), etc.

269 – Just keep your decimal places straight and this is simple.

285 – This is a good test of your understanding or odd (O) and even (E) operations. Begin by writing down how an odd can happen: O+O= , O×O=O, and O/O=O. The only equation that satisfies one of these is 2p + q = E+O = O. Case closed.

299 – Like all inequality problems, this one is tricky. You have two inequalities to work with. $0 < 1 - \frac{c}{d}$ and $1 - \frac{c}{d} < 1$. The first inequality gives $\frac{c}{d} < 1$ and the second one gives $\frac{c}{d} > 0$. The second inequality combined with d>0, gives c>0. Thus I and II are true. Testing option III is much more time consuming. Since c and d are positive integers such that c<d, we know that $c^2 < d^2$, but that is not enough to test III. Just follow the book method here and do a test to prove III must not be true.

351 – This is a good understanding of odd/even and integers. The question reminds us that 1. Zero is an integer. 2. Integers can be negative. 3. The positive and negative numbers must appear in equal pairs since the sum is zero.

360 – The issue here is getting to the answer fast. There are a number of shortcuts, but due to the numbers, none are especially fast. I new them and it still took me 3:36. First you need to find the cost of the largest pot. This is x+5(.25) = x+1.25. The total equation can be immediately written in terms of the first and last terms as:

$$6\left(\frac{F+L}{2}\right) = 3\left(x + (x + 1.25)\right) = 6x + 3.75 = 8.25$$

Solve for x=0.75 then solve for price of largest pot = .75+1.25. The test does the problem in negatives which seems much harder to me.