

*This test has 120 points; your score will be based upon 100. The extra points can be used to offset low scores on other tests. You can use one sheet of notes. **GOOD LUCK.***

- (6) In a presidential election poll, 1000 people were in favor of Kerry, and 1150 people were in favor of Bush. What are the odds for Bush winning the election?  
**1150:1000 or 23:20** Express your answer in the form X:1 **1.15:1**. What percentage of the votes do you expect Kerry to get in the final election? **46.5%**
- (10) A sample space is defined by rolling a single die. If n equals the number of dots on the die, the sample space is defined to be the numbers  $X = (n+1)n/2$ . (a) What is the sample space S? (b) What is the probability of getting a number divisible by 3? (c) What is the expected value of X?

a. (6) Sample Space: <b>1 3 6 10 15 21</b>
b. (2) P(divisible by 3) = <b>4/6</b>
c. (2) E(X) = <b>(1+3+6+10+15+21)/6 = 9.3</b>

- (14) A bag contains three yellow balls with the numbers 1, 2, 3 and two red balls with the numbers 4, 5. We are going to draw two balls **WITH** replacement. The outcomes are defined to be the color of the balls and the sum of numbers on the balls. What are the items in the sample space S? Two events A and B are defined as follows:  
 $A = \{YY, \text{ i.e. 2 yellow balls}\}$ ,  $B = \{\text{sum} \geq 5\}$

**Sample Space (2 Points) |S| = 25    |A| = 9    |B| = 19**

	Y1	Y2	Y3	R4	R5		
Y1	YY2	YY3	YY4	YR5	YR6		
Y2	YY3	YY4	-YY5-	YR6	YR7		
Y3	YY4	-YY5-	-YY6-	YR7	YR8		
R4	RY5	RY6	RY7	RR8	RR9		
R5	RY6	RY7	RY8	RR9	RR10		

Based upon the above sample space, compute the following probabilities (2 pts each):

P(A)	9/25	$P(A \cup B)$	25/25=1
P(B)	19/25	$P(A B)$	3/19
$P(A \cap B)$	3/25	$P(B A)$	3/9

4. (10) Given the following frequency table, generate a probability distribution (8) and compute the expected value (2) of X.

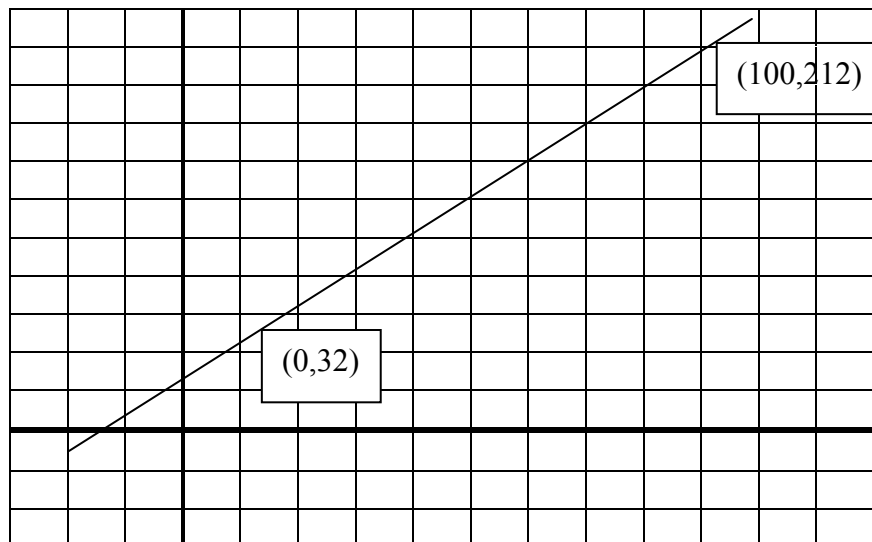
X	F(X)	P(X)	E(X)
3	10	10/24	30/24
5	8	8/24	40/24
7	4	4/24	28/24
9	2	2/24	18/24
$\Sigma$	24	24/24	116/24

$E(X) = 116/24 = 9.3$
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5. (10) Define the following terms.

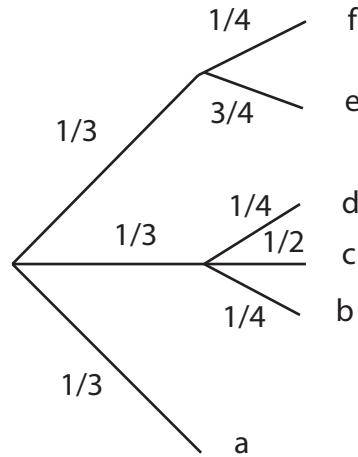
(5) Probability distribution: A set of numbers which are greater than zero and less than one which all add up to one. $0 < P_i < 1, \Sigma P_i = 1$
(5) Random variable: A variable which varies according to a probability distribution.

6. (5) Given the following pair of points: (0,32) and (100,212), what is the y-intercept equation for the straight line?  $Y = (9/5)X + 32$ . Draw a graph of the line showing both the x and y intercepts. You should plan before you draw!



7. (12) Using the following probability tree diagram, if  $G = \{a,c\}$  and  $H = \{c,d,e\}$ , what are the following probabilities. Show answers as fractions.

P(a)	4/12	P(G)	6/12
P(b)	1/12	P(H)	6/12
P(c)	2/12	P( $G \cup H$ )	5/6
P(d)	1/12	P( $G \cap H$ )	1/6
P(e)	3/12	P( $G H$ )	1/3
P(f)	1/12	P( $H G$ )	1/3



8. (13) If you roll a die four times and all the dots add up to 8, how many sets of numbers allow this to happen (1)? 5. For each set of numbers write the number ways each set of numbers could be arranged to make a total of eight. What is the probability of rolling the dice four times and getting at total of eight (2).

1115	4!/3!	4	
1124	4!/2!	12	
1133	4!/2!2!	6	
1223	4!/2!	12	
2222	4!/4!	1	
	Sum	35	
	P(8 on 4 rolls)	$35/6^4 =$	35/1296

9. (5) Show that  $\binom{n}{k} = \binom{n}{n-k}$  or equivalently show that  ${}_n C_k = {}_n C_{(n-k)}$

$$\binom{n}{n-k} = \frac{n!}{(n-k)!(n-(n-k))!} = \frac{n!}{(n-k)!(n-n+k)!} = \frac{n!}{(n-k)!k!} = \binom{n}{k}$$

10. (5) Show that  $k! \times {}_n C_k = {}_n P_k$

$$k! \times {}_n C_k = \frac{k!n!}{k!(n-k)!} = \frac{n!}{(n-k)!} = {}_n P_k$$

11. (10) Counting (2 pts each). You can leave your answer in factorial notation.
- How many ways can you arrange the following letters: DUGOUTS?
  - Using the following letters: DGOUTSR, how many different three letter words can you make that begin and end with a consonant and have a vowel as the middle letter? Letters can be used more than once. The words don't have to be real words.
  - There are four roads between towns A and B. There are six roads between towns B and C. How many different routes can one travel between A and C?
  - How many ways can 10 people be seated on a bench if only 4 seats are available?
  - How many ways can you select three students from a class of 40?

a	b	c	d	e
$7!/2!$	$5 \times 2 \times 5$	$4 \times 6$	$10!/(10-4)!$	$40!/3!(40-3)!$

12. (10) Compute the following (2 pts each). Show your work, but give answers as a single number.

$\frac{92!}{89!}$	${}_{12}P_3$	${}_{12}C_3$	$\frac{14!}{10!4!}$	$\binom{52}{2}$
753,450	1320	220	1001	1326

13. (10) Miscellaneous (2 pts each)

(2) The odds of Debra passing an exam are 12.5:1 in favor of passing. What is the probability of Debra passing?  $12.5/13.5 = .926$

(2) Bill just rolled four sixes in a row, what is the probability that he will roll another six on his fifth row?  $1/6$

(2) In the dice game of 10, what is the probability of busting on the second roll?  $3/26$

(2) If you are the first player in the dice game of 10, why is it advantageous to roll again when you have a total of 7? Since you have a 50/50 chance of busting it doesn't hurt you, but it forces the second player to bust more often.

(2) In the dice game of 10, you are the second player. The first player has rolled a total of 9. Now you have a total of 9. Should you roll again, why/why not? A guaranteed tie is better than a 5/6 chance of busting and loosing.