

Straight Line Problem

The boiling temperature of water decreases 3.4° for each 1000 meter increase in altitude. This relationship is given by the following equation:

$$B = -3.4 A + 100^\circ, \text{ where}$$

B is the boiling temperature of water at sea level in degrees Celsius

A is altitude in 1000s of meters

100° is the boiling temperature of water at sea level (0 meters of altitude).

To rewrite the equation for degrees Fahrenheit in altitude in 1000s of feet, we have two substitutions to make. First lets deal with altitude and convert 1000s of meters to 1000 of feet. One meter equals 3.281 feet or 1000 meters equals $3.281 * 1000$ feet. We can write this relationship as

$$km = \frac{kf}{3.281}, \text{ km is 1000s of meters and kf is 1000s of feet}$$

Rewriting our equation for B above in terms of 1000s of feet, we get

$$B = -1.036 kf + 100^\circ$$

Now we have to convert Celsius to Fahrenheit. We do this by substituting all of boiling equation into the Celsius to Fahrenheit conversion

$$F = 9/5 C + 32$$

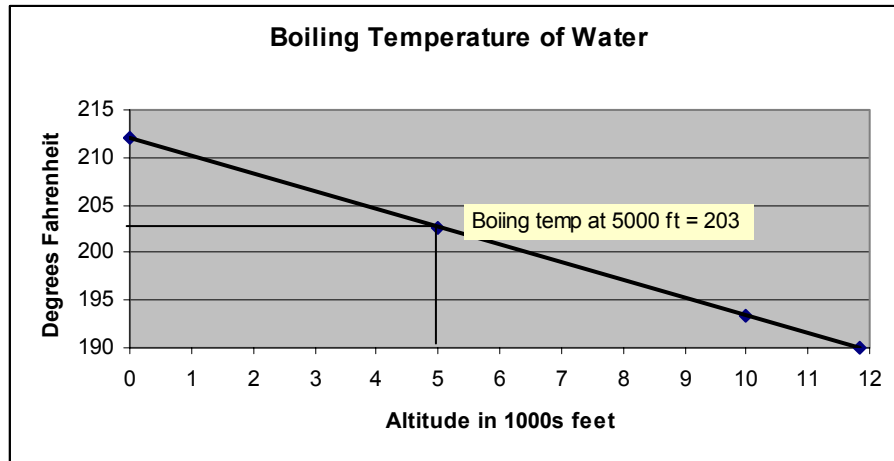
$$B = 9/5 (-1.036 kf + 100) + 32$$

$$B = -1.86 kf + 180 + 32$$

And our answer becomes

$$B = -1.86 kf + 212$$

Notice that 212° is the boiling temperature of water in Fahrenheit at sea level. The Result is graphed below.



The boiling temperature in Denver at 5000 feet in Fahrenheit is 203°

Odds Problem

1. The GMU basketball team is estimated to have a 30% chance of losing to Old Dominion U (ODU). What are the odds of GMU winning?
2. If the payoff for betting \$1 on GMU to lose is \$3, what is the expected payoff for a \$10 bet on ODU to win.
3. What is the expected amount of money you will lose if GMU wins?

The odds for GMU are 7:3 or you could write

$$\text{GMU:ODU} = 7:3$$

Since GMU is expected to win, we are going to bet on ODU where the payoff is higher, namely a \$1 bet pays off \$3. Betting for GMU to lose is the same as betting for ODU to win. It may not be nice to bet against your school, but you have a chance to win more money when you bet for a “long shot” winning. Since the payoff is 3 to 1 (not to be confused with the odds), a \$10 bet for ODU will payoff \$30 if ODU wins. If it helps you it might be easier to understand if you turn the odds around and have

$$\text{ODU:GMU} = 3:7$$

The expected amount of your payoff is given by:

$$\begin{aligned} \text{Expected Payoff} &= \text{Payoff} * P(\text{ODU win}) + 0 * P(\text{GMU win}) \\ \text{Expected Payoff} &= \$30(.3) + 0 (.7) \\ &= \$9 \end{aligned}$$

Expected winnings or expected loss is a combination of payoff and amount bet.

$$\begin{aligned} \text{Expected winnings} &= \$20 (.3) - \$10 (.7) \\ &= \$6 - \$7 \\ &= -\$1 \end{aligned}$$

That is, on average you expect to lose \$1 on every \$10 bet. Of course, you are only making one bet so there is no averaging for you, but for the hundreds of bets made by all persons, the bookie expects to make \$1 on average from all the bets placed. The bookie never loses because he has the averages working for him. You have no averages. You only have the chance of winning or losing according to the odds and payoff of the bet.

Probability Problem

A bag contains five balls. Three of the balls are red and have the numbers 1, 2, and 3 written on them. Two of the balls are yellow and have the numbers 7 and 8 written on them. You are going to draw two balls without replacement and add the numbers on the balls. Given that event $A = \{\text{both balls are red}\}$ and event $B = \{\text{sum of both balls is divisible by 3}\}$, Answer the following questions:

1. What is the sample space for this experiment?
2. $P(A)$
3. $P(B)$
4. $P(A \cup B) = P(A \cup B)$
5. $P(A \cap B) = P(A \cap B)$
6. $P(A|B)$
7. $P(B|A)$

The sample space looks like the following:

		Red	Red	Red	Yellow	Yellow
		1	2	3	7	8
Red	1		3	4	8	9
Red	2	3		5	9	10
Red	3	4	5		10	11
Yellow	7	8	9	10		15
Yellow	8	9	10	11	15	

Notice that the diagonals are blank because we are drawing without replacement; that is, once you have drawn a Red1 you cannot draw another one. Note that, there are only 20 possible outcomes (25-5), not 25, as you might first think.

$A = \{\text{both balls are red}\}$

$B = \{\text{sum of both balls is divisible by 3}\}$

P(A)	6/20	P(A ∩ B)	2/20
P(B)	8/20	P(A B)	2/8
P(A ∪ B)	12/20	P(B A)	2/6

Note: $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 6/20 + 8/20 - 2/20 = 12/20$

Extra Question

What is the expected value of the sum of the two balls or what is the average sum?

The average sum of two balls is 8.4

X	F(X)	P(X)	E(X)
3	2	0.1	0.3
4	2	0.1	0.4
5	2	0.1	0.5
8	2	0.1	0.8
9	4	0.2	1.8
10	4	0.2	2
11	2	0.1	1.1
15	2	0.1	1.5
	20		8.4