General Comments

This dice game turned out to be a better learning experience than I had expected, and a bit more complicated. The data for my games are summarized in an EXCEL spread sheet with accompanying charts. My summary sheet looks a little different from the one I put on the web page because, I wanted to record more data than I asked you to record. This wasn't a problem for me since I played the game on the computer by generating random numbers in the range of one to six rather than actually rolling the dice. I think after you look at it you will be able to figure it out.

Answers to Questions

1. The experimental and theoretical probability distributions and plotted individually and together in EXCEL. My experimental distribution did not agree with the theoretical distribution as well I expected. I believe this is because EXCEL does not do a good job of generating random numbers. I think rolling the dice would have been better. I will have to do some testing to verify this. I suspect your examples agreed better than mine.

2. To measure the expected number of rolls without busting, I wanted to get as many data points as possible so I rolled the dice five times, assuming I would bust by then, and counted how many times it took to bust. Out of 60 rolls, I only rolled five times twice without busting. The expected count after three rolls is 10.5, so you will have busted by the third or fourth roll most of the time. The expected number of rolls before busting for my data was 2.6. Of course you could roll one's ten times in a row, but that is very unlikely. In fact, it is $1/6^{10}$ or 1.65 E -8, a very small number.

3. The minimum number of rolls without busting is one, but it is always advantageous to roll at least two times. See number 4.

4. The optimum strategy for the first player is straight forward and depends on the probability of busting given that you have various totals.

Given	P(B|6,7,8)	P(Not BUST)	Action
6	1/3	2/3	Roll
7	1/2	1/2	Either, I chose to stay
8	2/3	1/3	Stay

Therefore with six, is always advantageous to roll. With seven, it doesn't matter. I chose to stay, but after thinking about it, I think it would be advantageous to roll on 7 since it would make it harder for the second player to beat you without busting. With 8 or above, it is always better to stay. See spread for details.

5. The first player always follows the rules above. The second has the advantage as described below.

6. It is always advantageous to be second for several reasons. First, if the first person busts, you automatically win, and you do not have to risk going busted yourself. Second, you know what you have to beat. You will bust more frequently than the first player, but some of those times you will win rather than going bust. An interesting subtlety is that if you tie on a number greater than seven, it is better to accept the tie rather than risk going bust and losing.

7. Given that the first player has nine, the only way you can win is to roll a ten. This is investigated in detail in question 8.

8. The number of times I got a ten on the second, third, fourth, and fifth rolls are summarized in the EXCEL spread sheets. I then calculated the number of time this could happen theoretically. It turned out to be a tricky problem and is summarized in immense detail in a separate spread sheet. Given that you got a ten, the expected number of rolls to get a ten is 3.02.

Bonus 1 – Average Total for n Dice

I also summarized the totals for three dice and derived theoretical values of getting various counts for three dice. These results are in a third EXCEL spread sheet.
It turns out if you total the amounts on three dice, you would expect to get 10.5. In general, if you roll n dice and total the dots, you can expect to get an average of n*3.5.

X = Total number of dots from n dice
E(X) = 3.5n

This is no surprise since the expected value for one roll is 3.5. This formula can be derived easily by noticing that the average total dots for n rolls is

(n+6n)/2 = 7n/2 = 3.5n

Bonus 2 – Sum of Opposite Faces

If you have an even sided die, why is it possible to always have the dots on opposite faces total to one more than the number of faces? The answer to this question uses the trick we used to sum the numbers from 1 to n. Consider the examples below.

	Six sided die		
Face	1	2	3
Opp	6	5	4
Sum	7	7	7

Ten sided die				
1	2	3	4	5
10	9	8	7	6
11	11	11	11	11

The sum of opposite faces is always equal to n+1, and the sum of all the faces is

$$\sum_{i=1}^{n} i = \frac{(n+1)n}{2}$$