Winning Door Probability Paradox

In September of 1991 a reader of Marilyn vos Savant's Sunday Parade column wrote in and asked the following question:

"Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the other doors, opens another door, say No. 3, which has a goat. He then says to you, 'Do you want to pick door No. 2?' Is it to your advantage to take the switch?"

The expected and most obvious answer is that there is no reason to switch since the probabilities change to ½, ½ from 1/3, 1/3, 1/3 after a door is exposed. The trick however, is that game show host knows where the car is and must expose a door without the car. Once you have made a choice, the host is restricted in terms of what doors he can open. The result is if you switch doors, you will win 2/3 of the time. If you remain with the door chosen, you will win 1/3 of the time.

The **BEST** way to understand problems like this is to enumerate all possible outcomes.

Car	1	1	1	2	2	2	3	3	3
Pick	1	2	3	1	2	3	1	2	3
Expose	2or3	3	2	3	1or3	1	2	3	1or2
Action to win	stay with 1	**switch to 1**	**switch to 1**	**switch to 2**	stay with 2	**switch to 2**	**switch to 3**	**switch to 3**	stay with 3

The problem is that the host must pick a goat and if you already picked a goat (which happens 2/3 times) he has only one choice in picking the goat. That means that 2/3 of the time you will win if you switch.

Said another way: If you did not pick the winning car (picked a goat), which happens 2/3 of the time, the host is forced to pick the specific door which has the other goat (you telling you that the car is in the door he did not pick. This happens 2/3 of the time. If you switch, you win 2/3 of the time.

Problem Variation

If the show host didn't know where the car was located and could accidentally expose the car allowing you to lose right away, then the probabilities would in fact change to ½, ½. Actually this is a much more completed set of events. We must add the additional rule that the host cannot expose the door you chose. There are now 27 possible outcomes to consider. Nine of these outcomes are excluded because the host is not allowed to pick the same door as you. These are indicated by an X in the "Expose" column. That leaves 18 possible outcomes. They are summarized in three tables below.

Car	1	1	1	1	1	1	1	1	1
Pick	1	1	1	2	2	2	3	3	3
Expose	X	2	3	1	X	3	1	2	X
Switch	-	stay	stay	lose	-	switch	lose	switch	-

Car	2	2	2	2	2	2	2	2	2
Pick	1	1	1	2	2	2	3	3	3
Expose	X	2	3	1	X	3	1	2	X
Switch	-	lose	switch	stay	-	stay	switch	lose	-

Car	3	3	3	3	3	3	3	3	3
Pick	1	1	1	2	2	2	3	3	3
Expose	X	2	3	1	X	3	1	2	X
Switch	-	switch	lose	switch	-	lose	stay	stay	-

One third of the time (6/18) you will lose when the door is exposed by the host. Given that you have not lost when the door is exposed, there are 12 remaining possibilities. Of these, six switches will win and six stays will win. Thus the probability of winning is 50/50 whether you stay or switch doors.