Equations of a straight line Dr. Kelly

Straight line problems

- Given an equation, plot the line
- Given the line plot, write the equation
- Given two points derive the equation
- Convert a word problem into an equation

A straight line is determined by

- Any two points, e.g. (0,b), (a,0), $(\bar{x}, \bar{y})$, (x_1,y_1)
- A slope and one point

Axes

- Think of the Y axis as standing up straight; it is the vertical axis
- Think of the X as cross, crossing the Y axis; it is the horizontal axis

Special Points

- (0,b), x=0, y=b, is called the y-intercept in the form y = mx + b
- (a,0), x=a, y=0, is the x-intercept in the form y = m(x-a)
- $(\bar{x}, \bar{y})$, the mid-point of a regression line

Slope

- Designated by "little m" in the equation y = mx + b
- Negative if y decreases as x increase
- Positive if y increases as x increases
- Rise over run
- $m = \frac{(y_1 - y_2)}{(x_1 - x_2)} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{rise}{run} = \frac{Change\_in\_y}{Change\_in\_x} = \frac{\Delta y}{\Delta x}$
- It doesn't matter if you do y_2-y_1 or y_1-y_2, of course the x's have to be done in the same direction.
- If you pick $(x_1-x_2) = 1$, then the slope becomes just (y_1-y_2)!

Standard (normal) forms

- y = mx + b, using point (0,b) (**y-intercept** form) ($b = y_1 - mx_1$)
- Rewrite this as y = m(x +b/m) and you get the x-intercept form, where a = -b/m
- y = m(x-a), using point (a,0) (**x-intercept** form) ($a = x_1 – y_1/m$)
- Rewrite this as y = mx – ma and you get the y-intercept form, where b = -ma
- $(y-y_1) = m(x-x_1)$, using point (x_1,y_1) (point-slope form)
- $(y - y_1) = \frac{(y_1 - y_2)}{(x_1 - x_2)}(x - x_1)$, same as above with a substitution for m (point-slope form)
- Using the general form cx + dy + e = 0, you get
- $y = -\frac{c}{d}x - \frac{e}{d}$ slope = -c/d, (0,b) = (0,-e/d)
- $y = -\frac{c}{d}\left(x + \frac{e}{c}\right)$ slope = -c/d, (a,0) = (-e/c,0)

Tips on Graphing Straight Lines

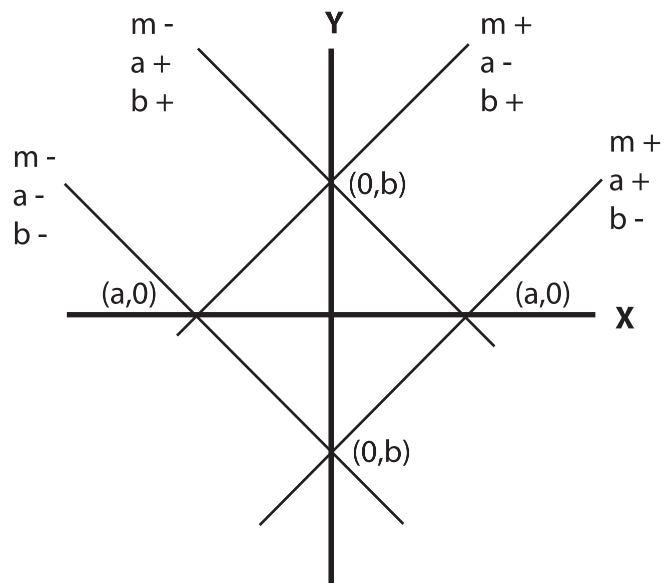
- ◊ It is easier to use two points rather than one point and a slope
- ◊ The best two points to use are (0,b) and (a,0)
- ◊ If you are graphing two arbitrary points make yourself a little table like the following:

This makes sure you take the Correct two pairs of points.
$(y_1-y_2) / (x_1-x_2)$

	X	Y
1	x_1	y_1
2	x_2	y_2

- ◊ Don't waste time finding more points except to check your results
- ◊ Scale your axes so as to show both x and y intercepts. (Sometimes this won't be possible.)

Four possibilities depending upon signs of the intercept values a and b



Four possible lines depending upon the signs of a and b

Parallel Lines

- ◊ $m_1 = m_2$ (slopes are equal)

Perpendicular Lines

- ◊ $m_1m_2 = -1$ (slopes are the negative reciprocal of each other)

Horizontal Line: y = a

Vertical Line: x = b

Examples

Situation	Equation
pay = (hourly wage) * (hours worked)	y = mx (intercept is 0)
bill = (price per item) * (number of items)	y = mx (intercept is 0)
cost = (fixed cost) + (variable cost)	y = b + mx (phone bills)
sales pay = (base pay) + (commission)	y = b + mx (comp plans)
feet = inches/12	y = x/12 (convert inches to feet
Fahrenheit = 9/5*(Celsius) + 32	y = 9/5 x + 32, m=9/5, b=32
Boiling temperature of water decreases 3.4° Celsius for each 1000 meter increase in altitude.	y = mx + b C_{boil} = -3.4A + 100
Air temperature decreases 6.5° F for each 1000 meter increase in altitude.	y = mx + b T = -6.5A + T_{surf}