Odds for and Against

This business of odds can be a very confusing topic, but is really just another version of probabilities stated differently. First and foremost, remember that it always concerns a binomial event: something happening versus something not happening, usually a win or lose situation. Odds are usually used for one time events, which will not be repeated again, at least under exactly the same conditions. The most common examples are horse races and sporting events like football and basketball. The odds are based on the past performance of the horse or team against similar competitors. Odds are much more difficult to calculate because there is no specific historical information for the exact event about to take place. The best you can do is look at past performance under similar situations. Odds on sporting events have no ware near the same precision as the odds on highly repeatable events like casino games.

> You can think of odds as the number of times team A would win (n) compared to the number of times team B would win (m), if they played each other n+m times.

In the following discussion, we will let A equal the event of something happening and $\overline{A}$ equal the event of A not happening. Consider the examples for A and $\overline{A}$ in Table 1.

Table 1 – Examples for Odds

A	$\overline{A}$	Experiment	P(A)	P($\overline{A}$)
A = {1}	$\overline{A}$ = {2,3,4,5,6}	Rolling a die	1/6	5/6
A = {H}	$\overline{A}$ = {T}	Flipping a coin	1/2	1/2
A = {Y}	$\overline{A}$ = {B}	Drawing from a bag contain 4 yellow and 1 black balls	4/5	1/5
A = {spade}	$\overline{A}$ = {H,D,C}	Drawing from deck	1/4	3/4
A = {win}	$\overline{A}$ = {lose}	Basketball game	1/3	2/3

Odds are always written in the form: **n : m,** where n and m can be counts or probabilities. That is,

$$\mathbf{n : m} \rightarrow |A| : |\overline{A}| \rightarrow \mathbf{P(A) : P(}\overline{A}\mathbf{)}$$

At first this seems confusing, but it becomes clearer when you substitute words for symbols

Outcomes in A : Outcomes in $\overline{A}$

Probability of A : Probability of $\overline{A}$

This is read as "the odds of A are n to m." In the case of rolling a "1" with a die is stated as "the odds of rolling a 1 are 1 to 5." Notice that the odds numbers are the numerators in the probability ratios 1/6 and 5/6. The odds can also be stated in terms of probabilities as "the odds of rolling a 1 are 1/6 to 5/6."

If the odds or A happening are n : m, then the probability of A can be written as

P(A) = n/(n+m)

$$P(A) = \frac{|A|}{|A| + |\overline{A}|} = \frac{|A|}{|S|}$$

where n = |A| and m = $|\overline{A}|$. The odds against A are just the opposite of the odds for A with n and m reversed:

Odds against A = m : n

$$|\overline{A}| : |A|$$

Basketball Odds

The odds for the college basketball championship were recently reported as

Team	Odds	Team	Odds
Arizona	9 : 5	Georgia Tech	3 : 1
Cincinnati	6 : 1	Gonzaga	2 : 1
Connecticut	4 : 5	Kentucky	3 : 1
Duke	1 : 1	Louisville	2 : 1

Consider the following examples:

What is the probability of Arizona winning?	9/15
What are the odds against Cincinnati?	1 : 6
What is the probability of Gonzaga loosing?	1/3

Expected Expected Winnings

If W is the amount you win over and above your bet, and B is the amount you bet, which is also the amount you lose if you lose, then the amount you can expect to win is equal to

Expected winnings = W * P(W) - B * P(L)

Of course, P(L) = 1 – P(W). Let's consider Georgia Tech in the above example. Georgia Tech has 3 : 1 odds in favor of winning or a probability of .75 of winning. If you bet a dollar and the amount you win over and above your bet is \$0.27, the your expected winning are

$$\text{Expected winnings} = \$0.27 * .75 - \$1.00 * .25 = -0.05$$

Expected Payoff

At the risk of being confusing, we could also look at the situation slightly differently by looking for the expected payoff, which can be written as

$$\textbf{Expected payoff} = (W+B) * P(W) + 0 * P(L)$$
$$= (W+B) * P(W)$$

And the expected winnings would be your expected payoff – your bet B. In our Georgia Tech example, this becomes

$$\text{Expected payoff} = \$1.27 * .75 = \$0.95$$

That is for every dollar you bet, you can expect to get back 95 cents over the long haul. Of course, there is no long haul since the basketball game is played only once. On your one bet, you would get back \$0 if Georgia Tech loses and \$1.27 if Georgia Tech wins. On the average, if 1000's of people bet for Georgia Tech, their average individual winnings would be \$0.95 for each dollar bet. That means that the bookmaker (the bookie) make 5% on all bets.

Now let's do some simple algebra and show how expected payoff and expected winnings are related.

$$\begin{aligned}\text{Exp win} &= \text{Exp pay} - B \\ &= (W + B)\,P(W) - B \\ &= W\,P(W) + B\,P(W) - B \\ &= W\,P(W) - B + B\,P(W) \\ &= W\,P(W) - B\,(1 - P(W)),\ \text{but } 1 - P(W) = P(L),\ \text{therefore} \\ &= W\,P(W) - B\,P(L)\end{aligned}$$

which is what was stated earlier.

Calculating a Profit Margin

Let's assume you are a bookie and you want to make 20% on all bets. After all, this is a risky business and you want to be handsomely rewarded. We'll continue with Georgia Tech. Since you expect Georgia Tech to win with 3:1 odds or 75% probability, you would be willing to payout \$0.80 for each \$1.00 bet, which give rise to the following payoff equation

$$.75\,P = \$0.80 \text{ or } P = \$1.07, \text{ where P is the payoff}$$