Expected Value – Another version of average

Consider the binomial experiment of flipping a coin three times. The sample space looks like the following

Event	X = # Heads
TTT	0
TTH	1
THT	1
HTT	1
HHT	2
HTH	2
THH	2
HHH	3
∑	12
X-bar	12/8

We know that the average is just the sum of the numbers (12) divided by the number of events in the sample space (8), which is 12/8. Now let's look at the summation from a different angle. Rather than adding up each individual value, we could shorten the process by looking at each different value and then multiple by the frequency of that value. In the above example, that would look like the following:

$$\begin{aligned} \text{Sum} &= 1(0) + 3(1) + 3(2) + 1(3) \\ &= 0 + 3 + 6 + 3 \\ &= 12 \end{aligned}$$

Now, we know the average is just the sum divided by the count or 12/8.

$$\begin{aligned} \text{Avg} = \bar{x} &= 12/8 \\ &= \frac{1(0) + 3(1) + 3(2) + 1(3)}{8} \end{aligned}$$

But that is the same as

$$= \frac{1}{8}(0) + \frac{3}{8}(1) + \frac{3}{8}(2) + \frac{1}{8}(3)$$

$$= P(0)0 + P(1)1 + P(2)2 + P(3)3$$

$$= p_1x_1 + p_2x_2 + p_3x_3 + p_4x_4$$

$$E(X) = \sum_{i=1}^{n} p_i x_i$$

This can all be summarized very nicely in the following table:

i	**x_i**	**$F(x_i)$**	**$P(x_i)$**	**$E(x_i) = x_i P(x_i)$**
1	0	1	1/8	0/8
2	1	3	3/8	3/8
3	2	3	3/8	6/8
4	3	1	1/8	3/8
		8	1.0	12/8

Notice that $F(x_i)$ is just what we have been calling count. We turn frequencies into probabilities by dividing individual frequencies (counts) by the total number of items in the sample space (total count).