The probability of two people A and B having the same birthday is the same as assuming that the BD of person A is fixed and asking what is the probability that the BD of person B is the same as that of A. Since there are 365 days in a year, that chance is 1/365. From this, we can easily conclude that the probability of them not having the same BD is 1-1/365.

Now let's back into this result from the reverse direction by asking, what is the probability that two people will not have the same BD? We already answered this, but by looking at it from the other direction, perhaps we will increase our understanding of what is going on. Once again, assume that the BD of A is fixed. We want to know the chances that B's BD does not match A's BD. For this to happen, B has to have a BD different from A. Since there are 364 remaining days (after A's birthday has been chosen) to chose from, there are 364/365 ways for B's BD not to match A's BD. From this we have to conclude that if the probability of not having the same birthday is 364/365, then the probability of having the same birthday must be 1-364/365. But we already said that this was 1/365. What's going on? Nothing, everything is as it should be. Some simple arithmetic shows that

$$1-\frac{364}{365}=\frac{365-364}{365}=\frac{1}{365}$$

We are now going to use this "back door" method of calculating the probability that at least one pair of people from a total of k people have the same BD. This result will follow from the following complementary relationship

P (at least one pair of BD's match in k people) = 1 - P (no BD's match in k people)

So let's proceed to find the probability that no BD's match and by default find the probability that at least one pair of people have a matching BD. If A and B are in the room without matching BD's and C walks in, the probability that C's BD does not match either A or B is 363/365. Two days have already been chosen by A and B, leaving 363 choices for C not to match either A or B. So far we have

$$P(no\_match)=\frac{365}{365}\frac{364}{365}\frac{363}{365}$$

This can be written as

$$P(no\_match)=\frac{365!}{365^3(365-3)!}$$

We can generalize this equation to k people as follows

$$P(no\_match)=\frac{365!}{365^k(365-k)!}$$

This is not a particularly efficient way to perform the computation. It can be described in the following iterative notation which is much more convenient for computation and is easily done in EXCEL.

$$\text{P(no match with k+1 people)} = \text{P(no match with k people)}\ \frac{365-k}{365}$$

For example with k=3, the probability of no match with four people is equal to the probability of no match with three people multiplied by 362/365.

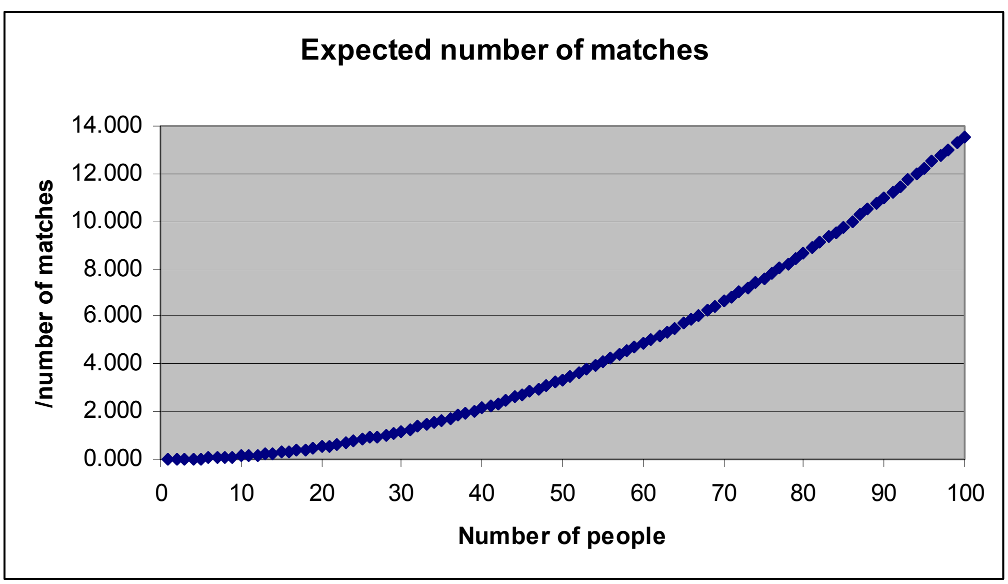
Now let's try to figure out how many pairs of matching BD's we can expect among k people. We know from our discussion on combinations that there are ${}_kC_2$ combinations of two things chosen from k things. Therefore, we have

$$_{k}C_{2} = \binom{k}{2} = \frac{k!}{2!(k-2)!} = \frac{k(k-1)}{2}$$

This is the number of possible pairs of people from a total of k people, and the probability that any one of them will have matching birthdays is 1/365, so the expected number of birthdays is

$$\frac{k(k-1)}{2\times 365} = \frac{k(k-1)}{730}$$

A graph of this equation for k varying between 1 and 100 is given below



k	Prob (no match)	P(at least one match)
1	1.000	0.000
2	0.997	0.003
3	0.992	0.008
4	0.984	0.016
5	0.973	0.027
6	0.960	0.040
7	0.944	0.056
8	0.926	0.074
9	0.905	0.095
10	0.883	0.117
11	0.859	0.141
12	0.833	0.167
13	0.806	0.194
14	0.777	0.223
15	0.747	0.253
16	0.716	0.284
17	0.685	0.315
18	0.653	0.347
19	0.621	0.379
20	0.589	0.411
21	0.556	0.444
22	0.524	0.476
23	0.493	0.507
24	0.462	0.538
25	0.431	0.569
26	0.402	0.598
27	0.373	0.627
28	0.346	0.654
29	0.319	0.681
30	0.294	0.706

k	Prob (no match)	P(at least one match)
31	0.270	0.730
32	0.247	0.753
33	0.225	0.775
34	0.205	0.795
35	0.186	0.814
36	0.168	0.832
37	0.151	0.849
38	0.136	0.864
39	0.122	0.878
40	0.109	0.891
41	0.097	0.903
42	0.086	0.914
43	0.076	0.924
44	0.067	0.933
45	0.059	0.941
46	0.052	0.948
47	0.045	0.955
48	0.039	0.961
49	0.034	0.966
50	0.030	0.970
51	0.026	0.974
52	0.022	0.978
53	0.019	0.981
54	0.016	0.984
55	0.014	0.986
56	0.012	0.988
57	0.010	0.990
58	0.008	0.992
59	0.007	0.993
60	0.006	0.994